7.1 Vectors in 2 Dimensions

Question Paper

Course	CIE A Level Maths
Section	7. Vectors
Topic	7.1 Vectors in 2 Dimensions
Difficulty	Very Hard

Time allowed: 70

Score: /52

Percentage: /100

Question 1a

A, B and C are the three vertices of a triangle. $\overrightarrow{AC} = 5\mathbf{i} - 2\mathbf{j}$ and $\overrightarrow{BC} = -3\mathbf{i} + k\mathbf{j}$, where k is a constant.

(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and k.

[2 marks]

Question 1b

(b) Given that $\left|\overrightarrow{AB}\right| = \sqrt{89}$, find the two possible values of k.

[3 marks]

Question 2a

(a) $\mathbf{a} = \begin{pmatrix} 8 \\ m \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} n \\ -2 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} m \\ n \end{pmatrix}$

Given that $\mathbf{a} + \mathbf{b} = \mathbf{c} - 2\mathbf{b}$, find the values of m and n.

[3 marks]

Question 2b

(b) $\mathbf{d} = \begin{pmatrix} 2k + 1 \\ 2k - 1 \end{pmatrix}$

Given that $|\mathbf{d}| = 3k\sqrt{2}$, find two possible values for k. Give your answer as an exact value.

[2 marks]

Question 3a

The point A lies on the circle with equation $(x - 11)^2 + (y - 7)^2 = 34$. A has position vector $\overrightarrow{OA} = 3k\mathbf{i} + 5k\mathbf{j}$, where k is a constant.

(a) Find the value of k, and hence determine the coordinates of A.

[4 marks]

Question 3b

(b) Explain why a line passing through O and A must be a tangent to the circle.

[2 marks]

Question 4

Points A, B and C have position vectors $\overrightarrow{OA} = -6\mathbf{i} - 2\mathbf{j}$, $\overrightarrow{OB} = \mathbf{i} + m\mathbf{j}$ and $\overrightarrow{OC} = 3\mathbf{i} - 8\mathbf{j}$, respectively.

Given that A, B and C lie on the same straight line, use a vector method to find the value of m.

[5 marks]

Question 5a

Vector $\overrightarrow{AB}$ has a magnitude of $2\sqrt{6}$ and makes an angle of $165°$ with the positive y-axis (measuring anticlockwise from the positive y-axis).

(a) Find $\overrightarrow{AB}$ in the form $a\mathbf{i} + b\mathbf{j}$, where both a and b are given as exact values.

[3 marks]

Question 5b

(b) Find a unit vector in the direction of $\overrightarrow{AB}$.

[2 marks]

Question 6a

A ship is searching for a radio buoy whose transmitter has ceased functioning. The ship sets out from point O and heads in the approximate direction of the buoy, travelling at a constant speed of 40 km/h in a direction parallel to the vector $\mathbf{i} + 3\mathbf{j}$. After travelling for ninety minutes the ship has reached point P. At that time, the ship receives a brief transmission from the buoy indicating that the buoy is at a bearing of $210°$ from the ship. The ship heads on that bearing at the same constant speed, and reaches the buoy at point Q in another 45 minutes. Given that vector $\overrightarrow{OQ} = x\mathbf{i} + y\mathbf{j}$ km, find the exact values of x and y.

(a) Given that vector $\overrightarrow{OQ} = x\mathbf{i} + y\mathbf{j}$ km, find the exact values of x and y.

[7 marks]

Question 6b

(b) How far was the buoy from the ship, and at what bearing, at the time the ship initially left point O? Give the distance in kilometers, and give your answers correct to 1 decimal place.

[3 marks]

Question 7a

In an experiment, three forces are acting on a particle. $\mathbf{F}_1 = 7\mathbf{i} - \mathbf{j}$ newtons and $\mathbf{F}_2 = x\mathbf{i} + y\mathbf{j}$ newtons are both constant forces, although the values of x and y are initially unknown. The third force is $\mathbf{F}_3 = k\mathbf{i} + k\sqrt{3}\mathbf{j}$ newtons, where $k \geq 0$ is a parameter that can be varied by the experimenters. The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$.

(a) Given that $\mathbf{R} = \mathbf{0}$ when the magnitude of $\mathbf{F}_3$ is 10 newtons, find the exact values of x and y.
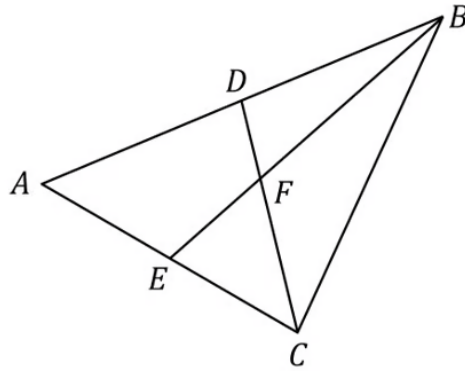
[4 marks]

Question 7b

(b) Find the magnitude of $\mathbf{F}_2$ and the angle it makes with the vector $\mathbf{i}$. Give your answers correct to 1 decimal place.

[3 marks]

Question 8a

In triangle ABC, D is the midpoint of AB and E is the midpoint of AC. BE and CD intersect at point F.

(a) Given that $\overrightarrow{AB} = 2\mathbf{a}$ and $\overrightarrow{AC} = 2\mathbf{b}$, write the vectors $\overrightarrow{BC}$, $\overrightarrow{BE}$ and $\overrightarrow{CD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[3 marks]

Question 8b

(b) By setting up and solving a suitable vector equations, prove that each of BE and CD divides the other in the ratio $1:2$

[6 marks]